Power optimization of a finite-time Rankine heat engine

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The power output of a single, finite-time Rankine heat engine is studied. The model adopted is a reversible Rankine cycle coupled to a heat source and a heat sink by heat transfer. Both the heat source and the heat sink have finite heat capacity rate. A mathematical expression is derived for the power output of the irreversible heat engine. The maximum power output is found. The maximum bound provides the basis for designing a real heat engine and for a performance comparison with existing power plants.

Keywords: heat engine; Rankine cycle; power output; finite-rate effects

Introduction

Among the important topics in thermodynamics has been the formulation of criteria for comparing the performance of real and ideal processes. Carnot showed that any heat engine absorbing heat from a higher-temperature reservoir to produce work must transfer some heat to a sink reservoir of lower temperature. He also showed that no engine could be better than the Carnot heat engine. The early tradition was carried on by Clausius, Kelvin, and others in using thermodynamics as a tool to find limits on work, heat transfer, efficiency, coefficient of performance, energy effectiveness, and figure of merit of energy conversion devices. The basic laws of thermodynamics were all conceived about irreversible processes. However, the subsequent development of thermodynamics has turned away from the process variables of heat and work toward state variables since Gibbs. The Carnot-Clausius-Kelvin view emphasizes the interaction of a thermodynamic system with its surroundings, while the Gibbs view makes the properties of the system dominant and focuses on equilibrium states. Contemporary classical thermodynamics gives a fairly complete description of equilibrium states and reversible processes. The only facts that it tells about real processes are that these irreversible processes always produce less work and more entropy than the corresponding reversible processes. Reversible processes are defined only in the limit of infinitely slow execution.

In the real engineering world, actual changes in enthalpy and free energy in an irreversible process rarely approach the corresponding ideal enthalpy and free energy changes. No practicing engineer wants to design a heat engine that runs infinitely slowly without producing power. There is a need to model an irreversible and time-dependent thermodynamic heat engine which can provide a power bound for designing a real heat engine.

The literature of finite-time thermodynamics started from Curzon and Ahlborn.[1] They treated a real Carnot engine power output being limited by the rates of heat transfer to and from the working substance. They showed theoretically that the heat engine efficiency at maximum power output is given by an expression different from the well-known Carnot efficiency, and they cited cases for which the efficiency of existing engines is well described by their result. Rubin[2] defined an endoreversible engine. In his simple model of the irreversible heat engine, all of the losses are associated with the transfer of heat to and from the engine and there are no internal losses within the engine itself. Because of the finite conductivity of the heat transfer material, the engine is operated not between the temperatures of the available high- and low-temperature heat reservoirs, T_H and T_L, but between the temperatures of the working fluid on the warm and cold sides of the heat engine cycle, T_W and T_C. The temperatures T_W and T_C depend on the rate of the heat flow and also on the power output of the machine. The efficiency of the engine also depends on its power output.

Andresen *et al.*,[3–5] Salamon *et al.*,[6] and Callen[7] developed thermodynamics in finite time to find the extremes for imperfect heat engines. A step Carnot cycle was defined and potentials for finite-time processes were constructed to determine the optimal performance of a real heat engine. Mozurkewich and Berry[8] studied optimization of a real heat engine based on a dissipative system. Band *et al.*[9] determined the optimal motion of a piston fitted to a cylinder containing a gas pumped with a given heating rate and coupled to a heat bath during finite times. Rubin[10] explored standards of performance for real energy conversion processes and reviewed the argument against the use of finitely slow reversible process standards. Rubin and Andresen[11] also found the optimal configuration for a class of heat engines with finite cycling times and suggested that figures of merit based on these optimal configurations may be more useful than those based on reversible processes. Rubin[12] then treated thermodynamic variables of the working fluid as dynamic variables and used mathematical techniques from optimal control theory to reanalyze the same class of irreversible heat engines as did Curzon and Ahlborn.[1] DeVos[13,14] generalized the linear heat transfer equation into a nonlinear form and described the results by a heat current flow and engine efficiency characteristics similar to the characteristic curve of a solar cell. Wu[15] has applied the cycle to an ocean thermal energy conversion system (OTEC). He[16] also extended the cycle to a cascade cycle and further modified the Curzon and Ahlborn simple finite-time heat engine model with both the heat source and the heat sink having finite-time heat capacity rate.[17]

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Finite-time Carnot heat engine and its power optimization

A practical heat engine is not as efficient as the classical Carnot heat engine. To achieve the theoretical Carnot cycle efficiency, the isothermal heating and cooling processes of the cycle must be carried out infinitely slowly to ensure that the working substance is in thermal equilibrium with its heat reservoirs. The power output of the cycle approaches zero since it requires an infinite time to get a finite amount of work. To obtain finite power, we must speed up the cycle. In the other extreme, if the heat engine speed were infinitely fast, the heat would flow directly from source to sink and no mechanical work would be performed by the heat engine. Hence, the power output and the heat engine efficiency would be zero. Somewhere between these two extremes, the heat engine has a maximum power output. The efficiency of the heat engine under the condition of maximum output has been evaluated experimentally.[17]

Let the heat engine cycle be made of two isothermal and two isentropic processes, as indicated in Figure 1. The cycle is a modified Carnot cycle with an irreversible isothermal expansion process from state 2 to state 3, an isentropic expansion process from state 3 to state 4, an irreversible isothermal compression cooling process from state 4 to state 1, and an isentropic compression process from state 1 to state 2. Process 2–3 is irreversible because heat flows from the high-temperature heat source reservoir to the working fluid at temperature T_W across a temperature difference, as illustrated schematically in Figure 1. Similarly, in the irreversible heat rejection process 4–1, heat flows across a temperature difference from the working fluid at a temperature T_C to the low-temperature heat sink reservoir.

We note that both the heat source and heat sink have finite heat capacity rates. Therefore, the temperature distributions of the heating fluid (heat source) and the cooling fluid (heat sink)

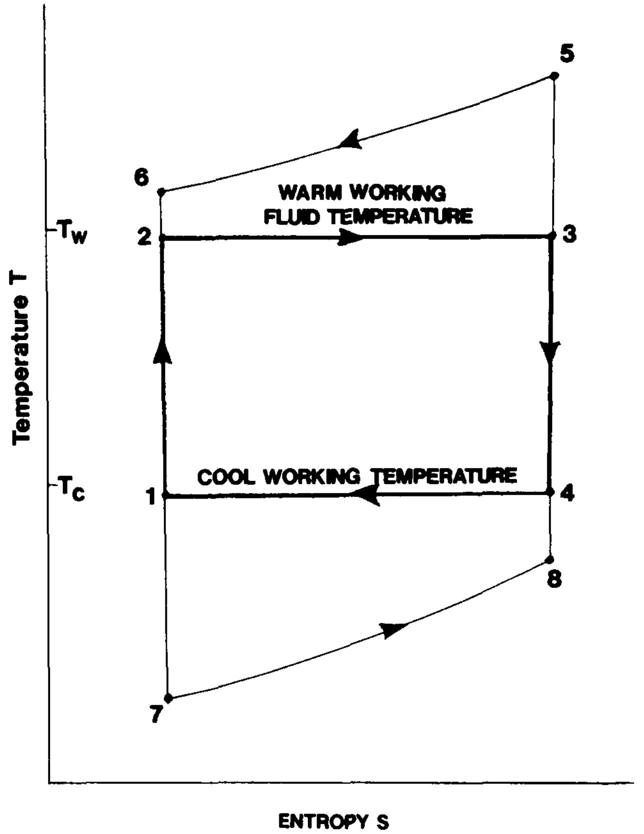

Figure 1 Finite-time Carnot cycle with heat source and sink having finite heat capacity rates

Notation

A_H	Surface area of the heat exchanger between the source and the heat engine
A_L	Surface area of the heat exchanger between the heat sink and the heat engine
a	Ratio of $U_H A_H$ to $U_L A_L$ defined in Equation 21
b	Defined in Equation 22
C	Constant defined in Equation 15
H	Total enthalpy
h	Specific enthalpy
$\mathrm{LMTD_H}$	Log mean temperature difference in the heat exchanger between the heat source and the heat engine
$\mathrm{LMTD_L}$	Log mean temperature difference in the heat exchanger between the heat sink and the heat engine
P	Power generated by the heat engine
P_{max}	Maximum power generated by the heat engine
Q_H	Heat transferred from the heat source to the heat engine
Q_L	Heat transferred from the heat engine to the heat sink
$\dot{Q}_H$	Rate of heat transfer from the heat source to the heat engine
$\dot{Q}_L$	Rate of heat transfer from the heat engine to the heat sink
S	Total entropy
s	Specific entropy
t	Total time required for the whole heat engine cycle
t_H	Time required to transfer Q_H
t_L	Time required to transfer Q_L
t_{12}	Time required for the isentropic pumping process
t_{34}	Time required for the isentropic expansion process
T	Temperature
T_C	Temperature of the working fluid in the condenser
T_H	Temperature of the heat source
T_L	Temperature of the heat sink
T_M	Mean effective temperature
T_W	Temperature of the working fluid in the boiler
T_5	Inlet temperature of the heating fluid
T_6	Outlet temperature of the heating fluid
T_7	Inlet temperature of the cooling fluid
T_8	Outlet temperature of the cooling fluid
u	$\mathrm{LMTD_H}$ defined in Equation 19
U_H	Overall heat transfer coefficient in the heat exchanger between the heat source and the heat engine
U_L	Overall heat transfer coefficient in the heat exchanger between the heat sink and the heat engine
v	$\mathrm{LMTD_L}$ defined in Equation 20
W	Output work generated by the heat engine
x	T_W
y	T_C
Z	P, defined in Equation 18
η	Thermodynamic cycle efficiency

are not constants throughout the heat exchangers, as shown in Figure 1.

The rate of heat flow from the high-temperature reservoir to the system is proportional to the log mean temperature difference $LMTD_H$. If t_H is the time required to transfer an amount Q_H of heat, then

$$\dot{Q}_H = Q_H/t_H = U_H A_H(LMTD_H) \quad (1)$$

where

$$LMTD_H = \frac{(T_5 - T_W) - (T_6 - T_W)}{\ln (T_5 - T_W)/(T_6 - T_W)}$$

U_H is the overall heat transfer coefficient including conduction, convection, and radiation modes; A_H is the surface area of the heat exchanger between the heat source and the system; and T_5 and T_6 are the inlet and outlet temperatures of the heating fluid of the heat source, respectively.

A similar expression holds for the rate of heat flow Q_L/t_L from the system to the low-temperature reservoir:

$$\dot{Q}_L = Q_L/t_L = U_L A_L(LMTD_L) \quad (2)$$

where

$$LMTD_L = \frac{(T_C - T_7) - (T_C - T_8)}{\ln (T_C - T_7)/(T_C - T_8)}$$

t_L is the time required to transfer the heat; U_L is the overall heat transfer coefficient; A_L is the surface area of the heat exchanger between the system and the heat sink; and T_7 and T_8 are the inlet and outlet temperatures of the cooling fluid of the heat sink, respectively.

The usual way to create an isentropic process is to pass the working fluid through the isentropic device so quickly that the system exchanges little heat with the surroundings. Therefore, the times required for the two isentropic processes, t_{34} and t_{12}, of the cycle are negligibly small relative to t_H and t_L.

The total time t required for the whole cycle is

$$t = t_H + t_L + t_{34} + t_{12} = t_H + t_L \quad (3)$$

where $t_{34} \ll t_H$, $t_{12} \ll t_H$, $t_{34} \ll t_L$, $t_{12} \ll t_L$. Since Q_H, Q_L, and the output work W are related to the Carnot heat engine operating between the temperatures T_W and T_C, Equation 3 becomes

$$t = Q_H(U_H A_H(LMTD_H))^{-1} + Q_L(U_L A_L(LMTD_L))^{-1}$$
$$= \frac{1}{U_H A_H} \frac{W}{LMTD_H} \frac{T_W}{T_W - T_C} + \frac{1}{U_L A_L} \frac{W}{LMTD_L} \frac{T_C}{T_W - T_C} \quad (4)$$

or

$$P = \frac{W}{t} = \left(\frac{1}{H_H A_H} \frac{1}{LMTD_H} \frac{T_W}{T_W - T_C} + \frac{1}{U_L A_L} \frac{1}{LMTD_L} \frac{T_C}{T_W - T_C}\right)^{-1} \quad (5)$$

where P is the power output of the irreversible heat engine. We consider the inlet and outlet temperatures of the heat source and heat sink (T_5, T_6, T_7, T_8) and the heat conductances ($U_H A_H$, $U_L A_L$) of heat exchangers are to be fixed. P is then a function of T_W and T_C only. Maximizing P with respect to the two as yet undetermined working fluid temperatures T_W and T_C yields

$$\partial P/\partial T_W = 0 \quad (6)$$

$$\partial P/\partial T_C = 0 \quad (7)$$

We solve Equations 6 and 7 numerically to obtain the optimum intermediate temperatures. Substituting these values into Equation 5 yields the optimum power delivered by the irreversible heat engine P_{max}.

It can also be shown that

$$\partial^2 P/\partial T_W^2 < 0 \quad (8)$$

$$\partial^2 P/\partial T_C^2 < 0 \quad (9)$$

Equations 8 and 9 verify that the power output of the irreversible heat engine is indeed the maximum, where P is the power output of the reversible heat engine. For a heat source and heat sink with infinite heat capacity, the temperature distributions of the heating and cooling fluids are constants throughout the heat exchangers, as shown in Figure 2. The case then simplifies to that of Curzon.[1] Equations 1, 2, and 5 become, respectively,

$$Q_H/t_H = U_H A_H(T_H - T_W) \quad (10)$$

$$Q_L/t_L = U_L A_L(T_C - T_L) \quad (11)$$

$$P = \frac{W}{t} = \frac{1}{U_H A_H} \frac{1}{T_H - T_W} \frac{T_W}{T_W - T_C} + \frac{1}{U_L A_L} \frac{1}{T_C - T_L} \frac{T_C}{T_W - T_C} \quad (12)$$

where $T_H = T_5 = T_6$ and $T_L = T_7 = T_8$.

The optimum intermediate temperatures are[1]

$$T_W = C(T_H)^{0.5} \quad (13)$$

$$T_C = C(T_L)^{0.5} \quad (14)$$

where

$$C = [(U_H A_H T_H)^{0.5} + (U_L A_L T_L)^{0.5}][(U_H A_H)^{0.5} + (U_L A_L)^{0.5}]^{-1} \quad (15)$$

Substituting Equations 13 and 14 into Equation 12 yields the optimum power delivered by the irreversible heat engine and the efficiency at optimum power.

$$P_{max} = U_H A_H U_L A_L (T_H^{0.5} - T_L^{0.5})^2 ((U_H A_H)^{0.5} + (U_L A_L)^{0.5})^{-2} \quad (16)$$

$$\eta = \frac{P_{max}}{Q_H/t_H} = 1 - \left(\frac{T_L}{T_H}\right)^{0.5} \quad (17)$$

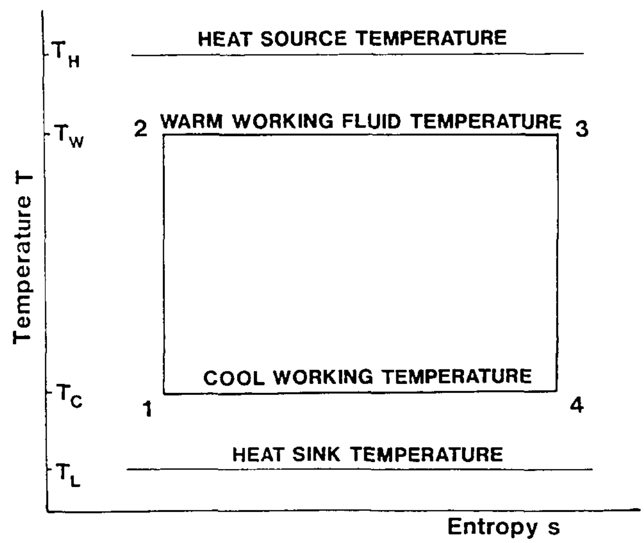

Figure 2 Finite-time Carnot cycle with infinite heat capacity rate for the heat source and sink

Curzon and Ahlborn[1] claimed that large power plants are operated closer to this efficiency than to the Carnot efficiency, and illustrated their claim by comparisons with a coal-fired steam-power plant, a nuclear reactor, and a geothermal steam-power plant.

Finite-time Rankine heat engine

The Carnot vapor cycle is not practical because of the difficulties encountered in carrying out some of the processes in actual devices. However, the difficulties with the Carnot vapor cycle may be eliminated by replacing the isothermal heat transfer processes with isobaric processes. We would have a finite-time Rankine engine consisting of two isentropic and two irreversible isobaric processes, as shown in Figure 3. Again, the finite-time Rankine cycle is a heat engine where possible irreversibilities can only take place in the heat transport and not in the conversion of the heat to power. Since the area under process 2–3 in the *T-s* diagram of Figure 2 represents the amount of heat added to the Rankine cycle, if we make this area equal to the area under a horizontal line (isothermal process) with a mean effective temperature of heat addition, T_M, we would have

$$T_M(S_3-S_2)=Q_H=H_3-H_2 \tag{18}$$

and

$$T_M=(h_3-h_2)/(s_3-s_2) \tag{19}$$

where S and H are total entropy and enthalpy, and s and h are specific entropy and enthalpy, respectively.

The modified finite-time Rankine cycle becomes a finite-time Carnot cycle operating between T_M and T_1. Since T_1 is the same as T_C in Figure 1 while $T_M=T_3$ (T_W in Figure 1), the finite-time Rankine cycle is seen to have a lower thermal efficiency and output power than those of a finite-time Carnot cycle operating between the same temperature limits. To raise the mean effective temperature of heat addition, we need a superheater in the real Rankine cycle. An efficiency and power output analysis on the finite-time Rankine heat engine is performed in the following numerical example.

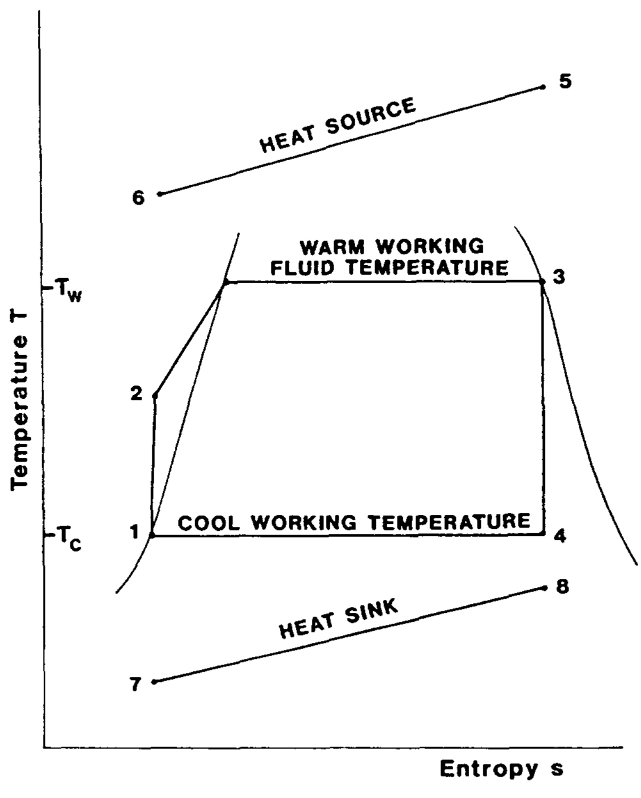

Figure 3 Finite-time Rankine cycle

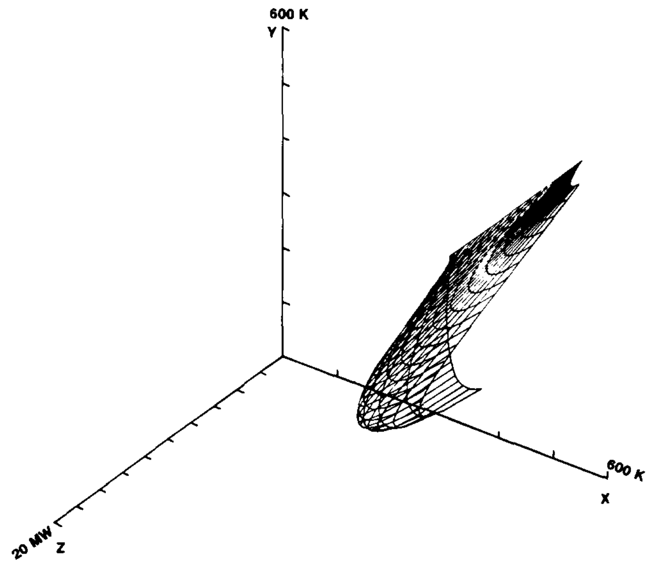

Figure 4 Power output surface ($Z=P$, $x=T_M$, $y=T_C$)

Numerical example

We take the inlet and outlet temperatures of the heating and cooling fluids to $T_5=700$ K, $T_6=529$ K, $T_7=293$ K, and $T_8=303$ K. Also let $U_LA_L=1$ MW/K, and $U_HA_H=0.03647$ MW/K.

$$P=(x-y)abuv/(xv+ayu)=Z \tag{20}$$

where

$$u=\frac{T_5-T_6}{\ln(T_5-x)/(T_6-x)}=\mathrm{LMTD_H} \tag{21}$$

$$v=\frac{T_8-T_7}{\ln(y-T_7)/(y-T_8)}=\mathrm{LMTD_L} \tag{22}$$

$$a=U_HA_H/U_LA_L \tag{23}$$

$$b=U_LA_L \quad x=T_W \quad y=T_C \tag{24}$$

P is a function of x and y. A typical plot of $P(x, y)$ is shown in Figure 4. The numerical solution for optimum power of a finite-time steam Rankine heat engine yields $T_W=458$ K, $T_C=319$ K, and $P_{max}=1.46$ MW, respectively. A practical engineer could match the temperatures (T_W, T_C) with a condenser pressure of 10 kPa (saturation temperature is 319 K) and a boiler pressure of 2 MPa (saturation temperature is 485 K). Referring to the states in Figure 3, we find their thermodynamic properties: $s_1=0.649$ kJ/kg K, $h_1=192$ kJ/kg, $v_1=0.00101$ m^3/kg, $s_3=6.34$ kJ/kg K, and $h_3=2800$ kJ/kg. The specific pump work, enthalpy at state 2, quality at 4, enthalpy at 4, turbine work, net work, heat added, and cycle efficiency can be calculated as $w_p=2$ kJ/kg, $h_2=194$ kJ/kg, $x_4=0.759$, $h_4=2007$ kJ/kg, $w_T=792$ kJ/kg, $w_N=790$ kJ/kg, $q_A=2605$ kJ/kg, and $\eta=0.303$, respectively. Notice that the mean effective temperature of heat addition, $T_M=(h_3-h_2)/(s_3-s_1)=458$ K, is equal to T_W exactly.

The efficiency of a finite-time Carnot heat engine operating between the same temperature limits of 319 K and 485 K is 0.5226.

Conclusion

An irreversible Rankine heat engine may be modeled by using an irreversibility factor and a time factor to simulate the primary heat transfer processes for the rate of energy exchange between the heat engine and its surroundings. This approach gives a very realistic prediction of heat engine efficiency. The power optimization process also provides a power bound for designing a real heat engine and for performance comparisons between existing heat engines.

References

1 Curzon, F. L. and Ahlborn, B. Efficiency of a Carnot engine at maximum power output. *Amer. J. Phys.* 1975, **43**, 22–24
2 Rubin, M. H. Optical configuration of a class of irreversible engine I. *Phys. RVW* 1979, **A19**, 1272–1276
3 Andresen, B., Salamon, P., and Berry, R. S. Thermodynamics in finite time extremals for imperfect heat engines. *J. Chem. Phys.* 1977, **66**, 1571–1577
4 Andresen, B., Salamon, P., Nitzan, A., and Berry, R. S. Thermodynamics in finite time. I. The step Carnot cycle. *Phys. RVW* 1977, **A15**, 2086–2093
5 Andresen, B., Salamon, P., and Berry, R. S. Thermodynamics in finite time. *Phys. Today* 1984, **37**(9), 62–70
6 Salamon, P., Andresen, B., and Berry, R. S. Thermodynamics in finite time. II. Potentials for finite-time processes. *Phys. RVW* 1977, **A15**, 2094–2102
7 Callen, H. B. *Thermodynamics and an Introduction to Thermostatistics*, 2nd ed., Wiley, New York, 1985
8 Mozurkewich, M. and Berry, R. S. Optimization of a heat engine based on a dissipative system. *J. Appl. Phys.* 1983, **54**(7), 3651–3661
9 Band, Y. B., K|fri, O., and Salamon, P. Finite time thermodynamics: Optimal expansion of a heated working fluid. *J. Appl. Phys.* 1982, **53**(1), 8–28
10 Rubin, M. H. Figure of merit for energy conversion process. *Am. J. Phys.* 1978, **46**, 637–639
11 Rubin, M. H. and Andresen, B. Optimal staging of endoreversible heat engine. *J. Appl. Phys.* 1982, **53**(1), 1–7
12 Rubin, M. H. Optimum configuration of a class of irreversible heat engine. II. *Phys. RVW* 1979, **A19**, 1277–1289
13 DeVos, A. Efficiency of some heat engines at maximum power conditions. *Am. J. Phys.* 1985, **53**, 570–573
14 DeVos, A. Reflections on the power delivered by endoreversible engines. *J. Phys. D: Appl. Phys.* 1987, **20**, 232–236
15 Wu, C. A performance bound for real OTEC heat engines. *J. Ocean Engrg.* 1987, **14**(4), 349–354
16 Wu, C. Optimal performance of a cascade endoreversible cycle. ASME Paper No. 87-WA/DSC-6, 1987
17 Wu, C. Power optimization of a finite-time Carnot heat engine. *J. Energy* 1988, **13**(9), 681–687